**Ice Module**

**General Input parameters**

= Ice thickness (m)

= Ice velocity (m/s)

= Wind turbine supporting structure diameter at contact node (m)

= Density of water (kg/m3)

= Ice mass density (kg/m3)

= Ice sheet initial location (m)

= Ice load starting time (s)

**Ice Module – ice model 1**

**Initialization input**

None

**Initialization output**

None

**Inputs (u)**

= displacement of the interaction node on wind turbine supporting structure (m)

= velocity of the interaction node on wind turbine supporting structure (m/s)

**Outputs (y)**

= Ice force (N)

**States**

*Continuous states* (**x**)

= displacement of the ice feature (m)

= velocity of the ice feature (m/s)

*Discrete States* (**x­­d**)

None

*Constraint States* (**z**)

None

**Input parameters**

= Indentation factor

= Ice crystal type factor (MPa-3•s-1)

= Activation energy (kJ•mol-1)

= Universal gas constant (J•mol-1K-1)

= Ice temperature (K)

= Poison ratio of ice

= Ice wedge angle (rad)

= Nominal ice stress (MPa)

= Elastic modulus of ice (Pa)

**Parameters (p)**

= Ice thickness (m)\*

= Ice velocity (m/s)\*

= Wind turbine supporting structure diameter at contact node (m)\*

= Ice load starting time (s)\*

= Indentation factor

= Elastic modulus of ice (Pa)

= Time when the maximum force is reached (s)

= Maximum ice force (N)

**Mathematical Formulation**

**Governing equations:**

None

**Continuous state function (**)**:**

None

**Discrete-State Functions ()**

None

**Constraint-State Functions ()**

None

**Output function ():**

Sub model 1 – Creep:

Sub model 2 – Elastic buckling:

Sub model 3 – Nominal stress

**Ice Module – ice model 2**

**Initialization input**

None

**Initialization output**

None

**Inputs (u)**

= displacement of the interaction node on wind turbine supporting structure (m)

= velocity of the interaction node on wind turbine supporting structure (m/s)

**Outputs (y)**

= Ice force (N)

**States**

*Continuous states* (**x**)

= displacement of the ice feature (m)

= velocity of the ice feature (m/s)

*Discrete States* (**x­­d**)

= Number of the current ice tooth

*Constraint States* (**z**)

None

**Input parameters**

= Ice brittle strength (MPa)

= Maximum ice tooth tip displacement (m)

= Distance between sequential ice teeth (m)

**Parameters (p)**

= Ice thickness (m)\*

= Ice velocity (m/s)\*

= Wind turbine supporting structure diameter at contact node (m)\*

= Ice sheet initial location (m)\*

= Maximum ice tooth tip displacement (m)

= Distance between sequential ice teeth (m)

= Equivalent elastic stiffness of the ice teeth (N/m)\*

**Mathematical Formulation**

**Governing equations:**

None

**Continuous state function (**)**:**

None

**Discrete-State Functions ()**

Sub model 1-3:

When the displacement of the current ice tooth

The current ice tooth breaks

**Constraint-State Functions ()**

None

**Output function ():**

Sub model 1:

Sub model 2 – Two teeth bending :

**Ice Module – ice model 3**

**Initialization input**

None

**Initialization output**

None

**Inputs (u)**

= displacement of the interaction node on wind turbine supporting structure (m)

= velocity of the interaction node on wind turbine supporting structure (m/s)

**Outputs (y)**

= Ice force (N)

**States**

*Continuous states* (**x**)

= displacement of the ice feature (m)

= velocity of the ice feature (m/s)

*Discrete States* (**x­­d**)

= Number of the current loading event

= Ice thickness of the current event (m)

= Ice velocity of the current event (m/s)

= Ice strength of the current event (N/m)

= Starting time of the current event (s)

= Time when the maximum force is reached for the current event (s)

= End time of the current event (s)

= Maximum ice force of the current event (N)

= Maximum ice tooth tip displacement of the current ice tooth (m)

= Distance between the current ice tooth and the next one (m)

= Equivalent elastic stiffness of the current ice tooth (N/m)

= Maximum ice tooth tip displacement of the next ice tooth (m)

= Distance between the next ice tooth and the one after that (m)

= Equivalent elastic stiffness of the next ice tooth (N/m)

= The sum of pitches of all broken ice teeth (m)

*Constraint States* (**z**)

None

**Input parameters**

= Mean value of random ice thickness (m)

= Variance of random ice thickness (m2)

= Mean value of random ice velocity (m/s)

= Variance of the random ice velocity (m2/s2)

= Mean value of ice loading event duration time. (s)

= Mean value of ice strength (MPa)

= Variance of ice strength (MPa2)

= Mean value of the random maximum ice tooth tip displacement (m)

= Variance of the random maximum ice tooth tip displacement (m2)

= Mean value of is the random distance between sequential ice teeth (m)

= Variance of the random distance between sequential ice teeth (m2)

**Parameters (p)**

= Ice thickness (m)\*

= Ice velocity (m/s)\*

= Wind turbine supporting structure diameter at contact node (m)\*

= Mean value of random ice thickness (m)

= Variance of random ice thickness (m2)

= Mean value of random ice velocity (m/s)

= Variance of the random ice velocity (m2/s2)

= Mean value of ice loading event duration time. (s)

= Mean value of ice strength (MPa)

= Variance of ice strength (MPa2)

= Mean value of the random maximum ice tooth tip displacement (m)

= Variance of the random maximum ice tooth tip displacement (m2)

= Mean value of is the random distance between sequential ice teeth (m)

= Variance of the random distance between sequential ice teeth (m2)

**Mathematical Formulation**

**Governing equations:**

None

**Continuous state function (**)**:**

None

**Discrete-State Functions ()**

When the end time of the previous loading event is reached

A new set of , , and are generated according to their distributions.

Sub model 3:

When the current ice tooth breaks, a new set of , and are generated according to their distributions.

**Constraint-State Functions ()**

None

**Output function ():**

Sub model 1:

Sub model 2:

Sub model 3 – Random dynamic model

**Ice Module – ice model 4**

**Initialization input**

None

**Initialization output**

None

**Inputs (u)**

= displacement of the interaction node on wind turbine supporting structure (m)

= velocity of the interaction node on wind turbine supporting structure (m/s)

**Outputs (y)**

= Ice force (N)

**States**

*Continuous states* (**x**)

= displacement of the ice feature (m)

= velocity of the ice feature (m/s)

*Discrete States* (**x­­d**)

= Number of the current ice tooth in each failure zone

*Constraint States* (**z**)

None

**Input parameters**

= Distance between sequential ice teeth in each zone (m)

= Number of failure zones along contact width

= Number of failure zones along contact height/thickness

= Ice structure contact profile mean value (m)

= Ice structure contact profile standard deviation (m)

= Ice failure strength within each failure zone (MPa)

= Ice teeth maximum elastic deformation (m)

**Parameters (p)**

= Ice thickness (m)\*

= Ice velocity (m/s)\*

= Wind turbine supporting structure diameter at contact node (m)\*

= Ice sheet initial location (m)\*

= Distance between sequential ice teeth in each zone (m)

= Equivalent elastic stiffness of the ice teeth (N/m)\*

= Ice teeth maximum elastic deformation (m)

= Ice structure contact profile initial location (m)

= Ice structure contact profile

= Number of failure zones

**Mathematical Formulation**

**Governing equations:**

None

**Continuous state function (**)**:**

None

**Discrete-State Functions ()**

When the displacement of the ice tooth of the ith failure zone

The current ice tooth breaks

**Constraint-State Functions ()**

None

**Output function ():**

Sub model 1:

Sub model 2 – Two teeth bending :

Sub model 3 – Random dynamic model

**Ice Module – ice model 5**

**Initialization input**

None

**Initialization output**

None

**Inputs (u)**

= displacement of the interaction node on wind turbine supporting structure (m)

= velocity of the interaction node on wind turbine supporting structure (m/s)

**Outputs (y)**

= Ice force (N)

**States**

*Continuous states* (**x**)

= displacement of the ice feature (m)

= velocity of the ice feature (m/s)

*Discrete States* (**x­­d**)

= angel between broken ice sheet and level waterline

*Constraint States* (**z**)

None

**Input parameters**

= slope angle of the cone (degree)

\* = flag that indicates upward or downward breaking cone: 0 – upward, 1 – downward.

= cone waterline diameter (m)

= cone top diameter (m)

= ride-up ice thickness (m)

= friction coefficient between structure and ice

= flexural strength of ice (MPa)

= limit strain

= limit strain rate (s-1)

**Parameters (p)**

= Ice thickness (m)\*

= Ice velocity (m/s)\*

= ice mass density (kg/m3)\*

= water mass density (kg/m3)\*

= slope angle of the cone (rad)

= cone waterline diameter (m)

= horizontal breaking force (N)

= vertical breaking force (N)

= breaking length (m)

= ride-up height (m)

= initial ride-up ice weight (N)

= broken ice weight (N)

**Mathematical Formulation**

**Continuous state derivative (**)**:**

**Discrete-State Functions ()**

**Constraint-State Functions ()**

None

**Output function ():**

(1) At *,*

Sub-model 1:

where

Sub-model 2:

where

(2) When , for both sub-models:

where

For sub-model 1:

For sub-model 2:

If

If

**Ice Module – ice model 6**

**Initialization input**

None

**Initialization output**

None

**Inputs (u)**

= displacement of the interaction node on wind turbine supporting structure (m)

= velocity of the interaction node on wind turbine supporting structure (m/s)

**Outputs (y)**

= Ice force (N)

**States**

*Continuous states* (**x**)

= displacement of the ice feature (m)

= velocity of the ice feature (m/s)

*Discrete States* (**x­­d**)

= flag to indicate if the ice floe has splitted (0 not splitted, 1 splitted)

*Constraint States* (**z**)

None

**Input parameters**

= ice floe width (m)

= ice floe length (m)

= ice crushing strength pressure-area relation constant

= ice crushing strength pressure-area relation order

= external driving force (MN)

= Fracture toughness of ice ( kNm^(-3/2) )

= non-dimensional splitting force

**Parameters (p)**

= ice mass density (kg/m3)\*

= ice thickness (m)\*

= ice floe initial velocity (m/s)\*

= wind turbine supporting structure diameter at contact node (m)\*

= ice crushing strength pressure-area relation constant

= ice crushing strength pressure-area relation order

= external driving force (N)

= splitting force (N)

= ice floe mass (kg)

**Mathematical Formulation**

**Governing equations:**

**Continuous state derivative (**)**:**

**Discrete-State Functions ()**

None

**Constraint-State Functions ()**

None

**Output function ():**

Splitting force: . When ***Y*** exceeds *Fsp*, ice force *Y* drops to 0.